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# ON THE DESIGN OF FLAW-TOLERANT SPACE ELEVATOR CABLES

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**ABSTRACT**

In this paper we discuss the recent news@nature "The space elevator: going down? Study shows that proposed carbon nanotube cables won't hold up" related to our recent findings on the role of defects in the carbon nanotube-based space elevator megacable.

***Keywords****: space elevator, megacable, nanotube, defect, strength*

## 1. INTRODUCTION

A *space elevator* [1] basically consists of a cable attached to a planet surface for carrying payloads into space. If the cable is long enough the centrifugal forces exceed the gravity of the cable, that will work under tension; for the Earth this critical length is of the order of a hundred of megameters [2]. The most critical component in the space elevator design is undoubtedly the megacable, which requires a material with very high strength-density ratio [2]. Carbon nanotubes are thus ideal candidates to build such a megacable, as suggested by their low density and huge strength, recently measured by nano-tensile tests [3,4].

An optimized megacable must have a uniform tensile stress profile rather than a constant cross-sectional area [2], with a taper-ratio, defined as the ratio between the maximum (at the geosynchronous orbit) and minimum (at the planet surface) cross-sectional area, which is an exponential function of the strength-density ratio. Accordingly, the megacable could in principle be built by any material with a sufficiently large taper-ratio. For example, for steel this value is $\sim 10^{33}$, whereas for carbon nanotubes it must theoretically be only 1.9. Thus, the feasibility of the space elevator seems to become only currently plausible [5,6] thanks to the discovery of carbon nanotubes. But basing the design of the megacable on the theoretical strength of a single carbon nanotube, as in the current proposal, is naïve [7], as recently emphasized in the following related news@nature, written by J. Palmer:

"**The space elevator: going down? Study shows that proposed carbon nanotube cables won't hold up.** Is it possible to make a cable for a space elevator out of carbon nanotubes? Not anytime soon, if ever, says Nicola Pugno of the Polytechnic of Turin, Italy. Pugno's calculations show that inevitable defects in the nanotubes mean that such a cable simply wouldn't be strong enough. The idea of a space elevator was popularized in science fiction, where writers envisioned a 100,000-kilometre-long cable stretching straight up from the Earth's surface and fixed in a geosynchronous orbit. Payloads, or tourists, would simply ascend the cable into low-Earth orbit, eliminating the need for rocket launches. When carbon nanotubes were discovered to have an incredibly high strength-to-weight ratio, researchers hoped they would take the idea out of fiction and bring it into reality. But Pugno argues that atomic-scale defects in the nanotubes would reduce the strength of such a giant cable by at least 70%. **Space ribbon.** Researchers think that the best shape for a space-elevator cable would be a ribbon, about a metre wide and as thin as paper. It would need to withstand at least 62 gigapascals (GPa) of tension. That's about as much as in the rope of a tug-of-war with more than 100,000 people on each side. Laboratory tests have shown that individual nanotubes can withstand an average of about 100 GPa, an unusual strength that comes courtesy of their crystalline structure. But if a nanotube is missing just one carbon atom, this can reduce its strength by as much as 30%. And a bulk material made from such tubes is even weaker. Most fibres made from nanotubes have so far had a strength much lower than 1 GPa. Recent measurements of high-quality nanotubes have found them to be missing one carbon atom out of every $10^{12}$ bonds; that's about one defect over 4 micrometres of nanotube length. Defects of two or more missing atoms are much more rare, but Pugno points out that on the scale of the space elevator they become statistically probable. Using a mathematical model that he has devised himself, and which has been tested by predicting the strength of materials such as nano-crystalline diamond, Pugno calculates that large defects will unavoidably bring a cable's strength below about 30 GPa. His paper has been posted to arXiv, and will appear in the July edition of the *Journal of Physics: Condensed Matter*. Pugno adds that even if flawless nanotubes could be made for the space elevator, damage from micrometeorites and even erosion by oxygen atoms would render them weak. So can a space elevator be made? "With the technology available today? Never," he says. **Never say never.** This comes in sharp contrast to claims made by Bradley Edwards, whose space elevator feasibility study for NASA and a subsequent book have made him the most frequent spokesman for the project. Edwards, who is president and founder of the Dallas-based company Carbon Designs, shrugs off the controversy, and says that with adequate fundinghe could make cables at or above the 62-GPa benchmark in just three years. He suggests that the key step is carefully spinning long nanotubes together in a close-packed way, which encourages cooperative frictional forces that make the strengths of individual nanotubes less crucial. Pugno counters that the larger defects critically weaken the cable, no matter what its construction. And lab efforts thusfar don't seem to inspire much optimism. Ray Baughman, director of the NanoTech Institute in Dallas, published a paper in *Science* last year demonstrating metre-long cables spun in a similar way to Edwards' preferred design. But these too had a strength well below 1 GPa. Baughman says Pugno's results aren't surprising. It has been known for decades that crystalline materials are sensitive to defects, and that they show a clear drop in strength with increasing size. But he adds that a solution may one day be found. The space elevator, he says, "won't happen in my lifetime, but I don't like to say never".

**Table 1: Defect sizes and shapes identification (*a,b*) and related flaw tolerant taper-ratios $l(a,b)$ according to the present analysis applied to nano-tensile tests [3,4]. In bold type are treated the 15 different tensile tests of single walled carbon nanotubes in bundle [3], whereas in underlined type the 19 tests on multi walled carbon nanotubes [4]. All the data are reported with the exception of the three smallest values of 0.11, 0.12 [4] and 0.13 [3], for which we would need for example adjacent vacancies (2*b*/*q*~1) in number 2*a*/*q*=75-89, 64-74 and 55-63 and related flaw tolerant taper-ratios of 267-437, 175-267 and 121-168 respectively.**

| $l(a,b)$ **2b/q** / **2a/q** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 | 1.90 |
| **1** | 2.48 | 2.37 | 2.25 | 2.18 | 2.13 | 2.09 | 2.07 | 2.05 | 2.03 | 2.02 | 2.01 | 1.90 |
| **2** | 3.04 | 2.91 | 2.73 | 2.59 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.21 | 2.19 | 1.90 |
| **3** | 3.61 | **3.48** | 3.25 | 3.04 | 2.87 | 2.74 | 2.64 | 2.56 | 2.49 | 2.44 | 2.39 | 1.90 |
| **4** | **4.20** | 4.06 | **3.79** | 3.53 | 3.30 | 3.12 | 2.97 | 2.86 | 2.76 | 2.68 | 2.61 | 1.90 |
| **5** | 4.82 | 4.67 | 4.36 | 4.04 | **3.76** | 3.53 | 3.34 | 3.18 | 3.05 | 2.94 | 2.85 | 1.90 |
| **6** | 5.46 | 5.31 | 4.97 | 4.60 | 4.26 | 3.97 | 3.73 | 3.53 | 3.36 | 3.23 | 3.11 | 1.90 |
| **7** | 6.14 | 5.98 | **5.60** | 5.18 | 4.78 | **4.44** | **4.15** | 3.90 | 3.70 | 3.53 | 3.39 | 1.90 |
| **8** | **6.86** | 6.68 | 6.28 | **5.80** | 5.34 | 4.94 | 4.59 | 4.30 | 4.06 | **3.85** | 3.68 | 1.90 |
| **9** | **7.61** | **7.43** | **6.99** | 6.46 | 5.94 | 5.47 | 5.07 | 4.73 | **4.44** | **4.20** | 3.99 | 1.90 |
| **10** | 8.40 | 8.21 | 7.74 | **7.16** | 6.57 | 6.04 | **5.58** | 5.19 | 4.85 | 4.57 | 4.33 | 1.90 |
| **11** | **9.24** | **9.03** | 8.53 | 7.89 | 7.25 | 6.65 | 6.12 | **5.67** | 5.29 | 4.96 | 4.68 | 1.90 |
| **12** | 10.12 | 9.90 | **9.36** | 8.67 | 7.96 | **7.29** | 6.70 | 6.19 | **5.75** | 5.38 | 5.06 | 1.90 |
| **13** | 11.04 | 10.81 | 10.23 | **9.49** | 8.71 | 7.97 | **7.31** | 6.74 | 6.24 | 5.82 | 5.46 | 1.90 |
| **14** | 12.01 | 11.77 | 11.16 | 10.35 | **9.50** | 8.69 | 7.96 | **7.32** | 6.76 | 6.29 | 5.88 | 1.90 |
| **15** | **13.03** | **12.78** | 12.13 | 11.26 | 10.34 | **9.45** | 8.65 | 7.93 | **7.31** | 6.78 | 6.32 | 1.90 |
| **16** | 14.10 | 13.84 | **13.15** | 12.22 | 11.23 | 10.26 | **9.37** | 8.58 | 7.90 | 7.31 | **6.80** | 1.90 |
| **17** | 15.23 | 14.95 | 14.22 | **13.23** | 12.16 | 11.11 | 10.14 | **9.27** | 8.51 | 7.86 | **7.29** | 1.90 |
| **18** | **16.41** | **16.11** | 15.34 | 14.29 | **13.14** | 12.00 | 10.94 | 9.99 | **9.16** | 8.44 | 7.82 | 1.90 |
| **19** | **17.65** | **17.33** | **16.52** | **15.41** | 14.17 | **12.94** | 11.79 | 10.76 | 9.85 | **9.06** | 8.37 | 1.90 |
| **20** | 18.94 | 18.62 | **17.76** | **16.57** | **15.25** | 13.93 | **12.69** | **11.57** | 10.57 | 9.71 | 8.96 | 1.90 |
| **21** | 20.30 | 19.96 | 19.05 | **17.80** | **16.39** | **14.97** | 13.63 | **12.41** | 11.33 | 10.39 | 9.57 | 1.90 |
| **22** | 21.72 | 21.36 | 20.41 | 19.08 | **17.58** | **16.06** | **14.62** | 13.31 | **12.14** | 11.11 | 10.22 | 1.90 |
| **23** | 23.21 | 22.83 | 21.83 | 20.43 | 18.84 | **17.21** | **15.66** | 14.24 | 12.98 | **11.87** | 10.90 | 1.90 |
| **24** | 24.76 | 24.37 | 23.32 | 21.84 | 20.15 | **18.42** | **16.76** | **15.23** | 13.86 | **12.66** | 11.61 | 1.90 |
| **25** | 26.39 | 25.98 | 24.88 | 23.32 | 21.52 | 19.68 | **17.90** | **16.26** | **14.79** | 13.49 | **12.36** | 1.90 |
| **26** | 28.08 | 27.66 | 26.50 | 24.86 | 22.96 | 21.00 | 19.10 | **17.35** | **15.77** | 14.37 | 13.14 | 1.90 |
| **27** | 29.86 | 29.41 | 28.20 | 26.47 | 24.47 | 22.38 | 20.36 | **18.49** | **16.79** | **15.29** | 13.97 | 1.90 |
| **28** | 31.71 | 31.24 | 29.97 | 28.16 | 26.04 | 23.83 | 21.68 | 19.68 | **17.87** | **16.25** | **14.83** | 1.90 |
| **29** | 33.64 | 33.15 | 31.83 | 29.92 | 27.68 | 25.35 | 23.06 | 20.93 | 18.99 | **17.26** | **15.74** | 1.90 |
| **30** | 35.65 | 35.14 | 33.76 | 31.75 | 29.40 | 26.93 | 24.51 | 22.24 | 20.17 | 18.32 | **16.68** | 1.90 |
| **31** | 37.75 | 37.22 | 35.77 | 33.67 | 31.20 | 28.59 | 26.02 | 23.60 | 21.40 | 19.42 | 17.68 | 1.90 |
| **32** | **39.93** | **39.38** | 37.87 | 35.67 | 33.07 | 30.32 | 27.60 | 25.03 | 22.69 | 20.58 | 18.71 | 1.90 |
| **33** | **42.21** | **41.63** | **40.06** | 37.75 | 35.02 | 32.12 | 29.25 | 26.53 | 24.03 | 21.79 | 19.80 | 1.90 |
| **34** | **44.58** | **43.98** | **42.34** | **39.92** | 37.06 | 34.00 | 30.97 | 28.09 | 25.44 | 23.05 | 20.93 | 1.90 |
| **35** | **47.05** | **46.42** | **44.71** | **42.19** | **39.18** | 35.97 | 32.76 | 29.72 | 26.91 | 24.37 | 22.12 | 1.90 |
| **36** | **49.61** | **48.97** | **47.18** | **44.54** | **41.39** | 38.01 | 34.64 | 31.42 | 28.44 | 25.75 | 23.36 | 1.90 |
| **37** | **52.28** | **51.61** | **49.75** | **46.99** | **43.69** | **40.14** | 36.59 | 33.19 | 30.04 | 27.19 | 24.65 | 1.90 |
| **38** | **55.06** | **54.36** | **52.42** | **49.54** | **46.09** | **42.36** | 38.62 | 35.04 | 31.71 | 28.69 | 25.99 | 1.90 |
| **39** | **57.94** | **57.21** | **55.19** | **52.20** | **48.58** | **44.68** | **40.74** | 36.97 | 33.45 | 30.26 | 27.40 | 1.90 |
| **40** | **60.94** | **60.18** | **58.08** | **54.96** | **51.18** | **47.08** | **42.95** | 38.97 | 35.27 | 31.89 | 28.86 | 1.90 |
| **41** | **64.05** | **63.27** | **61.08** | **57.82** | **53.87** | **49.59** | **45.25** | **41.06** | 37.16 | 33.59 | 30.39 | 1.90 |
| **42** | **67.28** | **66.47** | **64.19** | **60.80** | **56.68** | **52.19** | **47.64** | **43.24** | 39.12 | 35.36 | 31.98 | 1.90 |
| **43** | **70.64** | **69.79** | **67.43** | **63.89** | **59.59** | **54.90** | **50.13** | **45.51** | **41.17** | 37.21 | 33.64 | 1.90 |
| **44** | **74.12** | **73.24** | **70.79** | **67.11** | **62.62** | **57.71** | **52.72** | **47.86** | **43.31** | 39.13 | 35.36 | 1.90 |
| **45** | **77.73** | **76.82** | **74.27** | **70.44** | **65.76** | **60.64** | **55.41** | **50.31** | **45.52** | **41.13** | 37.16 | 1.90 |
| **46** | **81.48** | **80.53** | **77.88** | **73.90** | **69.03** | **63.68** | **58.20** | **52.86** | **47.83** | **43.21** | 39.03 | 1.90 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **47** | 85.36 | 84.38 | **81.63** | **77.49** | **72.41** | **66.83** | **61.10** | **55.51** | **50.23** | **45.37** | **40.97** | 1.90 |
| **48** | 89.39 | 88.37 | 85.52 | **81.22** | **75.93** | **70.10** | **64.12** | **58.26** | **52.72** | **47.61** | **42.99** | 1.90 |
| **49** | 93.56 | 92.51 | 89.55 | 85.08 | **79.57** | **73.50** | **67.25** | **61.12** | **55.31** | **49.95** | **45.09** | 1.90 |
| **50** | 97.88 | 96.79 | 93.72 | 89.08 | 83.35 | **77.02** | **70.50** | **64.08** | **58.00** | **52.38** | **47.27** | 1.90 |
| ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 1.9^(1+2*a/b*) |

Thus, is the elevator out of order?

## 2. ELEVATOR OUT OF ORDER?: FROM A NANO- TO A MEGA-TUBE

By considering (Dynamic) Quantized Fracture Mechanics [8-10], the ratio between the failure stress $s_N$ and its theoretical (defect-free) value $s_N^{(theo)}$ for a nanotube having atomic size $q$ (the “fracture quantum”) and containing an elliptical hole of half-axes $a$, perpendicular to the applied load (or nanotube axis), and $b$ can be determined including in the asymptotic solution [9] the contribution of the far field stress. Imposing the force equilibrium for a cable composed by a numerical fraction $f$ of defective nanotubes containing identical holes allows one to derive the cable strength reduction as a function of that of the nanotubes. Thus, a taper-ratio $l$ larger than its theoretical value (for carbon nanotubes $l^{(theo)} \approx 1.9$) would consequently be required for the megacable to be flaw tolerant against the propagation of an elliptical hole of half-axes $a$ and $b$. We find the results reported in Table 1. Designing the megacable with the theoretical taper-ratio, as in the current proposal [5,6], would surely lead to its failure [7]. In contrast, Table 1 can help in designing flaw-tolerant megacable.

In fact, defects are statistically expected in such huge a bundle [11]. At the thermal equilibrium the vacancy fraction $n/N << 1$ is estimated as $n/N \approx e^{-E_1/k_BT}$, where $E_1 \approx 7\text{eV}$ is the energy required to remove one carbon atom and $T$ is the absolute temperature at which the carbon is assembled, typically in the range between 2000 and 4000K. Thus, $n/N \approx 2.4\times10^{-18} - 1.6\times10^{-9}$. For the megacable, having a carbon weigh of ~5000Kg, the total number of atoms is $N \approx 2.5\cdot10^{29}$ thus a huge number of equilibrium defects, in the range $0.6\cdot10^{12} - 3.9\cdot10^{20}$ is expected, in agreement with a recent discussion [11] and observations [12]. Note that $n \propto N$, thus larger is weaker [13], i.e., further strength reductions are expected by increasing the size-scale from a nano- to a mega-tube, as recently discussed [7] and observed on meter-long carbon nanotube bundles [14].

By applying our treatment [8-10] to the results of each nano-tensile test on carbon nanotubes [3,4], we can identify plausible sizes and shapes for the most critical defect that has caused the nanotube fracture. Multiple nonlinear solutions clearly emerge; the results of our defect identification are reported in Table 1. Thus, assuming such plausible defects in the nanotubes composing the megacable, we deduce the corresponding flaw tolerant taper-ratios, see Table 1. Such huge taper-ratios, found to be two orders of magnitude larger than its theoretical value, are sufficient to place in doubt the effective realization of the Earth’s space elevator with the current technology and knowledge. Thus, assuming for the megacable the ideal strength of the nanotube, or the ideal taper-ratio, as in the current design [5,6], is naïve [7]: further studies on the role of defects are required [15], in parallel to the development of new technologies [16] capable of minimizing the thermodynamically unavoidable defects and align them along the megacable axis, as suggested by our results (see Table 1).

## 3. CONCLUSIONS

If the megacable will be designed assuming a strength of a defect-free nanotube, as in the current proposal [5,6], it will break [7]. Larger taper ratios, perhaps technologically unfeasible, are needed for a flaw tolerant design [15], as suggested by our calculations.


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