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ON THE COLLAPSE OF THREE TOWERS IN NEW YORK ON SEPTEMBER 11, 2001

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Abstract. The purpose of this paper is to estimate the floors where the New York collapses started from based on the well-documented evidence that all collapses took time equal to free fall time. An earlier analysis of the government-supported, progressive failure version showed that all collapses started significantly lower than the floors subjected to fire as a result of impact of terrorist planes, see Cherepanov and Esparragoza (2007) in Int. J. Fracture, V. 143, pp 203-206. A new, more sophisticated analysis of the New York collapses called the hybrid theory is advanced below. It explores the possibility that collapses started on several floors simultaneously, not in one floor as suggested in all previous theories. According to the hybrid theory the collapses of World Trade Center towers ran in two phases: (i) in the first phase dynamic fracture waves disintegrated a part of the towers producing the dust cloud and explosion sound well-documented and well-evidenced, and (ii) in the second phase the progressive failure front disintegrated the lower part of the towers that remained intact in the first phase. To illustrate many possible modes of progressive failure the slowest mode, the fastest mode, and three intermediate modes were first studied below and then the hybrid mode was introduced. The hybrid theory allows one to get rid of evident defects of earlier theories and explain all basic facts and observations of the matter not understood before, in particular, why the time of all collapses was free fall time independent of the position of critical floors the collapses started from. It was shown that the floors disintegrated in the very beginning of the collapses were located considerably lower than the floors hit by terrorists and subjected to fire. And so, this most sophisticated analysis of the New York collapses confirms the earlier conclusion and suggests that fracture waves were produced by explosives and not arisen from thermal stresses of fire as hypotesised earlier. The numerical work in the present paper was done by Prof. I. E. Esparragoza from Pennsylvania State University.

Keywords: Collapse, tower, WTC, mechanics, theory, evidence, observation, dust cloud, explosion, free fall, critical floor, fracture wave, progressive failure, the slowest mode, the fastest mode, intermediate mode, hybrid mode, analysis, calculations

1. Introduction

Progressive failure was the common mode of all collapses of tall buildings as emphasized by Bazant and Verdure (2006). In an earlier paper by Bazant and Zhou (2002) they expressed the belief of civil engineers of the world that progressive failure runs in the free fall regime, which seemed to have supported the government version of the New York collapses because of the well-known observation of the free fall time of all collapses.

According to the government version these collapses represented a simple progressive failure caused by the following chain of bad events: the crush of terrorist plane into a critical floor set a fire, severed 13% of the total of 287 steel columns and stripped fire insulation from all columns that collapsed in one hour from creep buckling, which triggered the start of general progressive collapse by buckling floor-by-floor up to the ground. Bazant and Verdure (2006) indicated that the towers would have survived if not all fire insulation had been stripped. National Institute of Standards studied column fragments in the rubble and established that all fire insulation had been stripped, but I think it was stripped during the collapse and not by terrorists.

The government theory of progressive failure proves to be inconsistent with all facts and observations on the initial stage of the collapses, see in this issue the previous paper by G.P. Cherepanov entitled "A critical analysis of the government theory: the collapse of the New York towers on September 11, 2001". However, on the final stage, it is well-confirmed by visual observations which show the progressive failure front marked by a dust cloud moving down into the intact structure.

An alternative theory by Cherepanov (2005) suggests that fracture waves disintegrated the towers at the very beginning of each collapse. The theory contradicts to the visual observation of the progressive failure front on the final stage of the collapses although it well explains dust, explosion sounds, and the free fall time of all three collapses. Also, in Cherepanov (2006a) it was proven that the acceleration of progressive failure is significantly less than the gravitational acceleration so that progressive failure is significantly slower than free fall which refutes the earlier belief of civil engineers that progressive failure is a free fall, see Bazant and Zhou (2002). [The report by Bazant and Verdure (2006) with a new unsuccessful attempt to explain free fall represents a response to the articles by Cherepanov (2006a and b) sent to Dr. Bazant in January, 2005].

Failure/fracture waves/ fronts are exposed and well-observed by dust clouds fracture waves create by pulverizing concrete, glass, marble etc. When we observe dust clouds in collapses we observe the work of fracture waves. A dust cloud is the marker of a fracture wave. Buckling or any other mode of failure cannot produce dust. When a car moves on a dusty road the car position is identified as the front of moving dust. As well the failure front in the tower collapse can be observed by dust clouds. According to the well-known pictures of the New York towers' collapses, a dust cloud was instantly created at the beginning of the collapses which covered a considerable part of the tower, with the lower part of the tower being intact. In a while, the lower front of dust cloud moved down marking the front of progressive failure.

Below we study five characteristic modes of progressive collapse and then introduce a hybrid mode that explains all available facts mentioned above.

2. Progressive Failure: Critical Floor Effect

Let us study progressive failure of a tower of height H_o. Very many different modes of progressive failure are possible depending, particularly, on the position of critical floor where a collapse starts from. Let us illustrate this point by five specific modes using the equations of progressive collapse derived by Cherepanov (2006a), with the rubble size being ignored. For the sake of simplicity and clarity let us assume also that the mass of the tower is uniformly distributed along the vertical and the resistance of the underlying, intact structure to the motion of the upper mass falling down is zero. These assumptions provide only a conservative estimate of the critical floors where collapses start from. The account of finite resistance and actual non-uniform mass distribution in the towers leads to the critical floors located significantly lower than those following from this estimate, because finite resistance and actual non-uniform mass distribution slow down the collapse process, see Cherepanov(2006), and Cherepanov and Esparragoza (2007).

a. The top floor is critical.

In this case the equations of progressive failure are as follows, Cherepanov (2006):

$$\frac{dM}{dt} = m\frac{dx}{dt}, \qquad \frac{d}{dt}\left(M\frac{dx}{dt}\right) = Mg \qquad \left(m = \frac{M_o}{H_o}\right). \tag{1}$$

Here: $g = 9.8$ m/s, t is time, x is the vertical coordinate positioning the front of progressive failure and directed downward so that the top of the tower corresponds to $x = 0$ and the ground floor to $x = H_o$, M is the mass of the upper structure that moves down under gravity force and increases with time because it absorbs the underlying structure, M_o is the mass of the whole tower, m is the mass of the tower per unit length assumed to be constant just for the purpose of getting some exact conservative estimate.

Under the natural initial conditions:

$$M = 0, \quad \frac{dx}{dt} = 0 \text{ when } t = 0, \tag{2}$$

the solution of equations (1) is very simple:

$$M = \frac{1}{6}mgt^2, \qquad x = \frac{1}{6}gt^2. \tag{3}$$

And so, in this mode the acceleration of the moving mass M is equal to $g/3$, and the collapse of the tower of height H_o, in meters, will take $\sqrt{6H_o/g}$ seconds. The height of the New York towers is $H_o = 420$ m; so that the collapse in this mode would take 16.4 s. It is the slowest mode of progressive failure. Of course, this time will be even less for the real value of mass distribution in the tower.

b. The ground floor is critical.

In this case referred below as mode B we have:

$$M = 2mx, \qquad \frac{d}{dt}\left(M\frac{dx}{dt}\right) = -Mg \quad \left(m = \frac{M_o}{H_o}\right). \tag{4}$$

Here: M is the mass of the intact structure that moves down under gravity force and decreases with time from M_o to zero because it is absorbed on the ground floor, $2x$ is the height of the intact structure at the moment of time t so that x is positioning its center of gravity. In this mode the ground floor coincides with the fixed front of progressive failure.

The initial condition equations in this mode are:

$$x = \frac{1}{2}H_o, \qquad \frac{dx}{dt} = 0 \text{ when } t = 0. \tag{5}$$

The solution to the problem (4) and (5) can be written as follows:

$$\frac{dx}{dt} = -\frac{1}{x}\sqrt{\frac{2}{3}g\left(\frac{1}{8}H_o^3 - x^3\right)},$$

$$t=\sqrt{\frac{3}{2g}}\int_{x}^{H_o/2}\frac{xdx}{\sqrt{\frac{1}{8}H_o^3-x^3}}.$$

And so, in this mode the acceleration of the moving mass M is equal to

$$\frac{d^2x}{dt^2}=-g\left[1+\frac{4m}{3x^2}\left(\frac{1}{8}H_o^3-x^3\right)\right]. \tag{7}$$

This acceleration is always greater than g and increases with decrease of x in the process of collapse.

The total time of collapse is equal to

$$T=\eta\sqrt{\frac{3H_o}{4g}}, \tag{8}$$

where

$$\eta=\int_0^1\frac{xdx}{\sqrt{1-x^3}}=\frac{\sqrt{\pi}\Gamma(2/3)}{3\Gamma(7/6)}\approx 0.85$$

in terms of Gamma- function $\Gamma(x)$.

The collapse of the New York towers in this mode would take $T = 4.8$ s. It is the fastest mode of progressive failure. Again, this time would be significantly greater for the real mass distribution in the tower.

For comparison, the collapse of a tall building of height H_o by industrial implosion takes $\sqrt{2H_o/g}$ seconds because it runs as free fall of disintegrated fragments. For the New York towers, it would take 9.4 s for any mass distribution in the towers, which is about the time observed for the collapses of both towers.

An intermediate floor is critical.

Suppose the collapse starts on a floor at height H_* from the ground so that the lower structure disintegrates on the progressive failure front moving down while the upper structure of height $(H_o - H_*)$ moves downward intact until the progressive failure front reaches the ground floor—the first phase; after that moment the upper, intact structure disintegrates in the fastest mode B studied above—the second phase.

For the first phase of the collapse we have:

$$\frac{dM}{dt}=m\frac{dx}{dt},\qquad \frac{d}{dt}\left(M\frac{dx}{dt}\right)=Mg\qquad \left(m=\frac{M_o}{H_o}\right). \tag{9}$$

When $t=0$: $x=0$, $M=M_*$ where $M_*=m(H_o-H_*)$.

Here: x is the vertical coordinate positioning the progressive failure front and directed downward, with the origin at the critical floor; M is the moving mass that increases from M_* to M_o on the first phase.

The solution to the problem (9) can be written as follows

$$\frac{dM}{dt} = \frac{1}{M}\sqrt{\frac{2}{3}mg}\sqrt{M^3 - M_*^3} \quad (M_* = m(H_o - H_*)), \tag{10}$$

$$t = \sqrt{\frac{3}{2mg}}\int_{M_*}^{M}\frac{MdM}{\sqrt{M^3 - M_*^3}}. \tag{11}$$

The acceleration of moving mass M is equal to

$$\frac{d^2x}{dt^2} = \frac{1}{3}g\left(1 + 2\frac{M_*^3}{M^3}\right). \tag{12}$$

And so, the time of collapse on this phase is equal to T_1

$$T_1 = \sqrt{\frac{3H_o}{2g}}\sqrt{\mu}F(\mu), \left(\mu = 1 - \frac{H_*}{H_o}\right) \tag{13}$$

where

$$F(\mu) = \int_{1}^{1/\mu}\frac{xdx}{\sqrt{x^3 - 1}}, \tag{14}$$

and

$$\lim_{\mu \to 0}\left[\sqrt{\mu}F(\mu)\right] = 2.$$

Function $F(\mu)$ decreases monotonically in interval (0, 1) with its derivative being infinite at $\mu = 0$ and $\mu = 1$. According to equation (12) the acceleration of moving mass M is equal to g at the beginning of the collapse and then decreases to $g/3$.

The upper structure of height $H_o - H_*$ can disintegrate in many different ways. I consider below only three pure modes of the second phase as follows:

(i) The upper structure instantly disintegrates within the time frame of the first phase or at the very beginning of the second phase;

(ii) In the end of the first phase when the failure front reaches the ground, the falling upper structure stops and then disintegrates on the bottom in mode B studied above. According to equation (8) the time of collapse on the second phase in this mode is equal to T_2

$$T_2 = \eta\sqrt{\frac{3}{4g}(H_o - H_*)}. \tag{15}$$

(iii) The upper structure moves down without to stop when the failure front reaches the ground, so that its speed in the end of the first phase is the same as at the beginning of the second phase. The second phase disintegration of the upper structure on the ground floor is governed by equations (4) under the following initial condition equations:

$$x = \frac{1}{2}(H_o - H_*), \qquad \frac{dx}{dt} = -V \text{ when } t = 0. \tag{16}$$

Here $t = 0$ is the beginning of the second phase and V is the speed of upper structure at the end of the first phase

$$V = \frac{1}{H_o}\sqrt{\frac{2}{3}g\left[H_o^3 - (H_o - H_*)^3\right]}, \tag{17}$$

as it follows from equations (9) and (10).

The solution of equations (4) satisfying equations (16) can be written as follows:

$$\frac{dx}{dt} = -\frac{1}{x}\sqrt{g\left(A - \frac{2}{3}x^3\right)}, \tag{18}$$

$$t = \frac{1}{\sqrt{g}}\int_x^{\mu H_o/2}\frac{xdx}{\sqrt{A - \frac{2}{3}x^3}}, \; M = 2mx \tag{19}$$

where

$$\mu = 1 - \frac{H_*}{H_o}, \tag{20}$$

$$A = \frac{1}{12}H_o^3\left[\mu^3 + 2\mu^2\left(1 - \mu^3\right)\right]. \tag{21}$$

The time of collapse on the second phase in this mode is equal to T_3

$$T_3 = \left(\frac{3}{2}\right)^{2/3} g^{-1/2}A^{1/6}G(\mu), \tag{22}$$

where

$$G(\mu) = \int_0^{a(\mu)}\frac{xdx}{\sqrt{1 - x^3}}, \tag{23}$$

$$a(\mu) = \left(\frac{\mu}{2 + \mu - 2\mu^3}\right)^{1/3}. \tag{24}$$

The total time of collapse is evidently equal to the sum of the durations of the first and second phases.

It is interesting to find the critical floor for which the total time of the collapse is the same as for free fall. Equating the total time of collapse in different modes to $\sqrt{2H_o/g}$, we can find the following equations:

$$\text{(i)} \; \sqrt{\mu}F(\mu) = \frac{2}{\sqrt{3}}, \tag{25}$$

$$\text{(ii)} \; \sqrt{3\mu}\left[\sqrt{2}F(\mu) + \eta\right] = 2\sqrt{2}, \tag{26}$$

$$\text{(iii)} \; \sqrt{\mu}F(\mu) + \frac{1}{\sqrt{2}}\mu^{1/3}\left(2 + \mu - 2\mu^3\right)^{1/6}G(\mu) = \frac{2}{\sqrt{3}}. \tag{27}$$

Here η, $F(\mu)$, and $G(\mu)$ are defined by equations (8), (14) and (23).

The root of equations (25) to (27) is as follows:

$$\text{Mode (i): } \mu = 0.31 \ \left(H_* / H_o = 0.69\right),$$
$$\text{Mode (ii): } \mu = 0.72 \ \left(H_* / H_o = 0.28\right), \tag{28}$$
$$\text{Mode (iii): } \mu = 0.35 \ \left(H_* / H_o = 0.65\right).$$

Applying these modes of progressive collapse to the New York twin towers predicts that the collapses of both towers started on the following floor:

The 75th floor assuming mode (i),
The 31st floor assuming mode (ii), and (29)
The 72nd floor assuming mode (iii).

It is assumed that the towers had 110 floors, 3.8 m high each floor.

Corresponding predictions for the neighboring 47-story building provide the following critical floor: (i) the 32nd floor, (ii) the 13th floor, and (iii) the 30th floor.

And so, the model of progressive failure is inconclusive if used accurately, although it always predicts that the critical floor of the collapse of the New York towers is significantly lower than the floors hit by terrorists and subjected to fire. It is a conservative estimate. An account of rubble size, real mass distribution, and resistance of underlying structure provides an even lower critical floor . An unsuccessful attempt by Bazant and Verdure (2006) to explain free fall time is due to that they used the 80th floor as the critical floor, which is too high, although it is still much lower than the 95th floor of the South Tower hit by terrorists.

Switching on and off different pure modes in the course of time may give an infinite number of various scenarios of collapse in the regime of progressive failure.

Also, other intact floors can get disintegrated, with forming some new zones of destruction, so that for many simultaneously acting critical floors the effect may be undistinguishable from that of fracture waves.

3. A Hybrid Mode

Any failure front is accompanied by some mini-scale fracture waves which pulverize concrete, glass, marble etc into dust. Therefore, dust clouds serve as some markers of failure fronts and fracture waves. The dust cloud observed in all collapses of the New York towers got almost instantly grown at the beginning of collapse and covered a considerable part of the tower. In a while, the lower front of the cloud moved downward along intact structure up to the ground. Based on this observation, the most reasonable scenario was this: (i) on the first phase fracture waves disintegrated a part of the tower immediately covered by a dust cloud and (ii) on the second phase the progressive failure front disintegrated the lower part of the tower remained intact on the first phase. Figure 1 depicts the fracture wave zone of length h_W almost instantly disintegrated on the first phase, and the progressive failure zone of length h_F which disintegrated on the second phase by progressive failure.

Let us calculate the time of collapse. We assume that the second phase started after top fragments in the fracture wave zone freely fell down on the top of the progressive failure zone which took $\sqrt{2h_W / g}$ seconds. For the second phase we have the following equations of progressive failure:

$$\frac{dM}{dt} = m\frac{dx}{dt}, \qquad \frac{d}{dt}\left(M\,\frac{dx}{dt}\right) = Mg - R\,, \tag{30}$$

$$x = 0, \qquad \frac{dx}{dt} = \sqrt{2gh_W}, \qquad M = M_P = m(H_o - h_F) \text{ when } t = 0. \tag{31}$$

Here: x is the vertical coordinate directed downward positioning the progressive failure front so that at the beginning of the second phase $t = 0$ and $x = 0$, and at the end of the second phase $t = T_3$ and $x = h_F$; M is the moving mass; R is the resistance of underlying, intact structure.

For the purpose of conservative estimate, we assume that $R = 0$. In this case, the solution to problem (30) and (31) can be written as follows:

$$\frac{dM}{dt} = \frac{1}{M}\sqrt{\frac{2}{3}mg}\sqrt{M^3 - M_H^3}, \tag{32}$$

$$t = \sqrt{\frac{3}{2mg}}\int_{M_P}^{M}\frac{MdM}{\sqrt{M^3 - M_H^3}}, \tag{33}$$

where

$$M_H^3 = 3m^3(H_o - h_F)^2\left(\frac{1}{3}H_o - \frac{1}{3}h_F - h_W\right). \tag{34}$$

The acceleration of moving mass M is equal to

$$\frac{d^2x}{dt^2} = \frac{1}{3}g\left(1 + 2\frac{M_H^3}{M^3}\right). \tag{35}$$

And so, the time of collapse on this phase is equal to T_3

$$T_3 = \sqrt{\frac{3}{2mg}}\int_{M_P}^{M_o}\frac{MdM}{\sqrt{M^3 - M_H^3}} \tag{36}$$

Here, evidently, $M_o > M_P > M_H \geq 0$.

The total time of collapse in the hybrid mode is equal to

$$T = \sqrt{\frac{2h_W}{g}} + \sqrt{\frac{3}{2mg}}\int_{M_P}^{M_o}\frac{MdM}{\sqrt{M^3 - M_H^3}}. \tag{37}$$

According to the actual observations the total time of collapses was equal to free fall time $\sqrt{2H_o / g}$ which provides the following equation

$$2\sqrt{H_o} = 2\sqrt{h_W} + \sqrt{\frac{3}{m}}\int_{M_P}^{M_o}\frac{MdM}{\sqrt{M^3 - M_H^3}}. \tag{38}$$

It serves to determine the parameters h_W and h_F defining the position of floors disintegrated by fracture waves at the very beginning of collapses.

4. An Analysis of the Hybrid Mode

Let us introduce the following dimensionless parameters f and δ

$$f = \frac{M_P}{M_o} = 1 - \frac{h_F}{H_o}, \qquad \delta^3 = f^3\left(1 - \frac{3h_W}{fH_o}\right), \tag{39}$$

So that

$$M_P = fM_o, \qquad M_H = \delta M_o. \tag{40}$$

From equations (34) and (39) it follows that

$$1 > f > \delta \geq 0. \tag{41}$$

Using f and δ equation (38) can be written as follows

$$\frac{3}{2}\int_f^1 \frac{xdx}{\sqrt{x^3 - \delta^3}} + \sqrt{f\left[1 - \left(\frac{\delta}{f}\right)^3\right]} = \sqrt{3}. \tag{42}$$

Let us start from the extreme case when $\delta = 0$. In this case we have:

$$\delta = 0, \qquad f = 3\frac{h_W}{H_o}, \quad \text{i.e.,} \qquad H_o = h_F + 3h_W, \tag{43}$$

$$\frac{dM}{dt} = \sqrt{\frac{2}{3}mgM}, \qquad t = \sqrt{\frac{6}{mg}}\left(\sqrt{M} - \sqrt{M_P}\right), \qquad \frac{d^2x}{dt^2} = \frac{g}{3}, \tag{44}$$

$$T_3 = \sqrt{\frac{6H_o}{g}}\left(1 - \sqrt{f}\right), \qquad T = \sqrt{\frac{2h_W}{g}} + \sqrt{\frac{6H_o}{g}}\left(1 - \sqrt{f}\right). \tag{45}$$

In this case equation (42) yields

$$\sqrt{f} = \frac{1}{2}\left(3 - \sqrt{3}\right), \qquad \text{i.e.,} \qquad f = 0.402 \tag{46}$$

From equations (43) and (46) it follows that

$$h_W = 0.133H_o, \qquad h_F = 0.598H_o. \tag{47}$$

Applying this result to the New York twin towers, we get that fracture waves disintegrated all floors from the 66th to the 81st floor at the very beginning, if this extreme case took place. According to equation (44) it is the slowest mode of the hybrid collapse. For all other modes the value of h_W, the wave-destructed zone, is smaller than 15 floors.

The numerical analysis of equation (42) results in the following table:

δ	0	0.1	0.2	0.3	~0.315
f	0.402	0.400	0.389	0.345	~0.316
h_W / H_o	0.133	0.131	0.112	0.040	0.0
h_F / H_o	0.598	0.600	0.610	0.655	0.684
Initially destructed floors of New York towers	66 to 81	66 to 81	67 to 80	72 to 77	~75

As seen, in the other extreme case when $\delta = 0.3150$ and $f = 0.316$ the hybrid mode coincides with the progressive failure mode (i) studied above in Section 2.

The present analysis shows that, similarly to the pure progressive collapse modes, the floors of New York towers destructed at the very beginning of the collapses appear to be significantly lower than the floors hit by terrorists and subjected to fire. From here, it follows that fracture waves were, probably, produced by explosives and not arisen from thermal stresses of fire as it was suggested earlier, see the previous paper of this author.

An account of resistance R of underlying, intact structure to the motion of falling mass leads to an increase of the wave-destructed zone h_W and a decrease of the initially intact zone h_F .It is remarkable that the official theory of pure progressive collapse, if correctly used, also predicts that the critical floor was located significantly lower than the floors hit by terrorists and subjected to fire. Unfortunately, this conclusion is under taboo in "free" scientific press .

And so, the hybrid mode allows us to also solve one of the mysteries of these collapses, namely, why the time of collapses was almost the same for both twin towers and almost equal to free fall time. Fracture waves scattered the effect of the position of the critical floor where the collapses started from, so that although this position may well be different in both towers, the collapse time is almost invariant with respect to this difference.

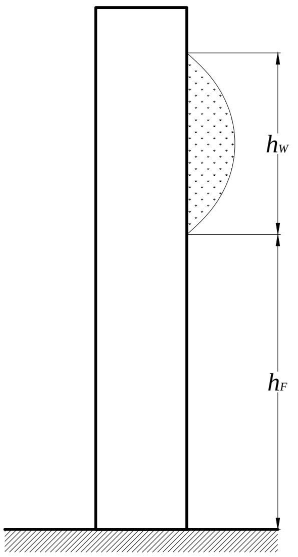

Fig. 1. Fracture wave zone of length h_W and progressive failure zone of length h_F .

5. Conclusion

It was shown that, if progressive failure would be the only regime of the collapses of the New York towers, then the time of collapses of both twin towers would be equal to free fall time, namely 9.4 s only if the critical floor, one and same for both towers, would be significantly lower than the floors hit by terrorists and subjected to fire. The latter conclusion follows also from the hybrid theory.

The hybrid theory was shown to explain the observations not understood earlier, namely (i) free fall time of all collapses, almost independent of the position of the critical floor, (ii) visual observations of the towers during the collapses that the lower parts of towers remained intact until failure front reached them, and (iii) visual observations of dust clouds, the markers of fracture waves. It proves that all collapses were initiated on the floors located significantly lower than the floors hit by terrorists and subjected to fire. The latter conclusion follows also from the equations of pure progressive collapse , see Bazant and Verdure (2006) , and Cherepanov and Esparragoza (2007).

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