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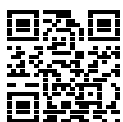
СИНГУЛЯРНАЯ ЗАДАЧА КОШИ ДЛЯ НЕЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ДРОБНОГО ПОРЯДКА

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Аннотация. Как показывают публикации, в различных областях задачи приводят к математическим моделям, основанным как на линейных [1,2], так и нелинейных [3-8] дифференциальных уравнениях. В случае нелинейных дифференциальных уравнений следует учитывать важный аспект нелинейности, наличие подвижных особых точек алгебраического типа у решения такого вида уравнений. Такие уравнения относятся к классу в общем случае не разрешимых в квадратурах. На данный момент, на основании публикаций, можно отметить два подхода в решении нелинейных дифференциальных уравнений. Первый подход связан с возможностью разрешить такие уравнения в квадратурах, если эти уравнения обладают свойством Пенлеве (наличие простых и кратных полюсов), что возможно лишь в частных случаях [9-19]. Второй подход связан с поиском аналитического приближенного решения нелинейных дифференциальных уравнений как обладающих свойством Пенлеве, так и не обладающих этим свойством (наличие критических полюсов) [20-28]. На основе исследования движения жидкости в трещиноватых коллекторах [30-32], дается постановка сингулярной задачи Коши для нелинейного дифференциального уравнения с дробной производной Герасимова – Капуро. Учитывая классификацию Фукса особых точек на неподвижные и подвижные [29], на первом этапе реализуется авторский метод [25] для неподвижной особой точки. Доказывается теорема существования и единственности решения в окрестности неподвижной особой точки. Получена априорная оценка погрешности аналитического приближенного решения и проведен численный эксперимент с графической интерпретацией.

Ключевые слова: нелинейное дифференциальное уравнение, дробная производная, особая точка, аналитическое приближенное решение, априорная погрешность.

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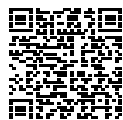
SINGULAR CAUCHY PROBLEM FOR A NONLINEAR FRACTIONAL DIFFERENTIAL EQUATION

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Abstract. According to the publications, problems solved in various fields lead to mathematical models based on both linear [1,2] and nonlinear [3-8] differential equations. In the case of nonlinear differential equations, an important aspect of nonlinearity must be taken into account: the presence of moving algebraic singular points in the solution of this type of equation. Such equations belong to the class of equations that are generally not solvable by quadratures. Currently, based on publications, two approaches to solve nonlinear differential equations can be distinguished. The first approach is associated with the possibility of solving such equations in quadratures if these equations possess the Painlevé property (the presence of simple and multiple poles), which is possible only in special cases [9-19]. The second approach is associated with looking up an analytical approximate solution to nonlinear differential equations, both those possessing the Painlevé property and those not possessing this property (the presence of critical poles) [20-28]. Based on the study of fluid flow in fractured reservoirs [30-32], a formulation of the singular Cauchy problem for a nonlinear differential equation with a Gerasimov–Caputo fractional derivative is given. Taking into account Fuchs’s classification of singular points as fixed and moving ones [29], the author’s method [25] is implemented at the first stage for a fixed singular point. A theorem for the existence and uniqueness of a solution in the neighborhood of a fixed singular point is proved. An a priori estimate of the error of the analytical approximate solution is obtained, and a numerical experiment with graphical interpretation is conducted.

Keywords: nonlinear differential equation, fractional derivative, singular point, analytical approximate solution, a priori error.

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ДОПОЛНИТЕЛЬНО

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ADDITIONAL INFORMATION

Competing interests. The authors declare that they have no competing interests.

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